MATHEMATICS WITHOUT BORDERS

AGE GROUP 8

SPRING 2020

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. Calculate a, if

$$-1-\frac{1}{1-\sqrt{2}}=\sqrt{2}+a.$$

Problem 2. If x_1 and x_2 are roots of the equation $x^2-3x-7=0$, calculate

$$|x_1-x_2|.$$

Problem 3. Calculate $m+n$, if $25x^{3m+2n}-x^{9-n}$ is a monomial (only has 1 term).

Problem 4. Calculate $(x-4)^3+(x-5)^4$, if $x^2-9x+20=0$

Problem 5. Find the smallest integer n such that $n\times\left(11-\sqrt{122}\right)<-1$.

Problem 6. Exactly one pair of brackets is to be inserted into the expression

$$\sqrt{2}\times\sqrt{2}-\sqrt{2}\times\sqrt{2}-\sqrt{2}\times\sqrt{2}-\sqrt{2}\times\sqrt{2}-\sqrt{2}\times\sqrt{2}.$$

The left bracket must be placed before a $\sqrt{2}$ and the right bracket after a $\sqrt{2}$. Find the greatest possible value of the resulting expression.

Problem 7. For how many integers x is the following inequality correct?

$$\frac{x+1}{\sqrt{-x+2}}\geq 0?$$

Problem 8. Write down the equation of a straight line which is perpendicular to the x-axis and has a point with coordinates (2020, 2021) on it.

Problem 9. Let a and b be the integer part and the fractional part of $\sqrt{6}$, respectively. Calculate the integer part of $a\div b$.

Problem 10. If

$$\sqrt{a^2-4a+5}+\sqrt{b^2-6b+10}=2$$

calculate $a+b$.

Problem 11. For which natural number x is the number equal to $(125^4)^x\times(4^{10})^3$ made up of 61 digits?

Problem 12. How many prime factors does the number equal to

$$3 + 3^2 + 3^3 + \ldots + 3^7 + 3^8$$

have?

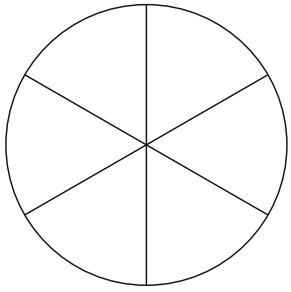
Problem 13. Find the total number of ways that 39 can be expressed as a sum of consecutive natural numbers.

Problem 14. The product of two negative numbers is 121, and their sum is S. How many possible integer values of S are there, greater than (-50)?

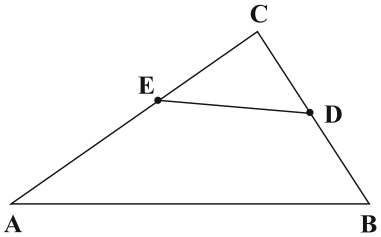
Problem 15. The natural number x is such that both x and x + 77 are perfect squares. Find the sum of all such natural numbers x.

Problem 16. The circle below is divided into six equal parts. We can colour one or more of these parts in black. How many different figures can be formed, in which there is at least one part coloured in black?

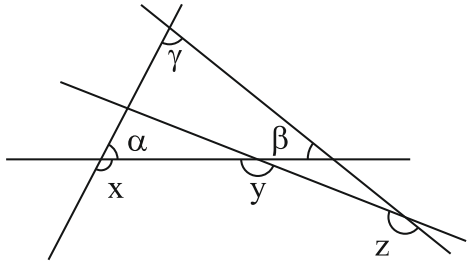
Please note: If you rotate the circle and end up getting the same figure, this does not count as a separate figure.



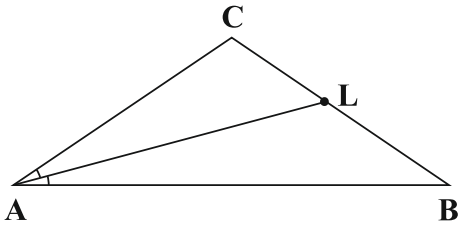
Problem 17. In triangle *ABC*, *AC* = 4 *cm* and *BC* = 5 *cm*. *D* is a point on *BC* and *CD* = 3 *cm*. *E* is a point on *AC*. The area of triangle *CED* is $\frac{1}{5}$ of the area of triangle *ABC*. Find the length of *CE* in *cm*.



Problem 18. If angle γ is equal to the average of the angles α and β, calculate the degrees of $x + y + z$.



Problem 19. *ABC* is an isosceles triangle and *AC* = *BC*. The point L lies on BC such that the line segment AL bisects $\angle CAB$. If $AC + CL = AB$, find $\angle CBA$.



Problem 20. The side lengths of the rectangle *ABCD* are 3 *cm* and 4 *cm*. The points *P* and *Q* are placed along the sides *BC* and *CD*, respectively, such that the area of ΔPQA is 3 cm^2.
Find the least possible value of *BP* + *DQ* in *cm*.